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DISTRIBUTED SYSTEM OPTIMAL CONTROL AND PARAMETER ESTIMATION: COMPUTATIONAL TECHNIQUES USING SPLINE APPROXIMATIONS

by H. T. Banks

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 82-0513	2. GOVT ACCESSION NO. AD-A117266	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DISTRIBUTED SYSTEM OPTIMAL CONTROL AND PARAMETER ESTIMATION: COMPUTATIONAL TECHNIQUES USING SPLINE APPROXIMATIONS		5. TYPE OF REPORT & PERIOD COVERED INTERIM
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H. T. Banks		8. CONTRACT OR GRANT NUMBER(s) AFOSR-81-0198
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lefschetz Center for Dynamical Systems Division of Applied Mathematics Brown University, Providence, RI 02912		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A4
11. CONTROLLING OFFICE NAME AND ADDRESS AFOSR/NM Bolling AFB, DC 20332		12. REPORT DATE April 1982
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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DISTRIBUTED SYSTEM OPTIMAL CONTROL AND PARAMETER ESTIMATION: COMPUTATIONAL TECHNIQUES USING SPLINE APPROXIMATIONS*†

by

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April 1982

LCDS #82-6

*Invited Lecture, 3rd IFAC Symposium on Control of Distributed Parameter Systems, June 29-July 2, 1982, Toulouse, France.

†This research supported in part by the Air Force Office of Scientific Research under contract AFOSR 81-0198, in part by the National Science Foundation under grant NSF MCS 7905774-05, and in part by the U. S. Army Research Office under contract ARO-DAAG29-79-C-0161. Parts of the research discussed were carried out at the Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, VA, which is operated under NASA contracts no. NAS1-15810 and No. NAS1-16394.

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ABSTRACT

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DISTRIBUTED SYSTEM OPTIMAL CONTROL AND PARAMETER ESTIMATION: COMPUTATIONAL TECHNIQUES USING SPLINE APPROXIMATIONS

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Abstract. Spline-based computational procedures for parameter estimation and optimal control problems involving delay and partial differential equations are outlined. Brief discussions are presented reporting on application of of the ideas to compute feedback controls for delay systems, to estimate variable coefficients in population dispersal models, and to estimate parameters in higher order models arising in elasticity.

Keywords. Parameter estimation; feedback controls; delay and distributed systems; computational methods; splines; wind tunnels; ecology; elasticity.

INTRODUCTION

In this note we present a brief summary of some computational methods that have proven useful in optimal control (both open loop and feedback) and parameter estimation problems for certain distributed systems. We first outline the basic ideas which are common to our approach whether we are dealing with functional differential equations (FDE) or partial differential equations (PDE) of hyperbolic or parabolic type. Roughly speaking, in each case one views the system under consideration as an abstract system

$$\dot{z}(t) = \mathcal{A}(q)z(t) + F(q,t)$$
$$z(0) = z_0 \tag{1}$$

in an appropriately chosen Hilbert space Z. In the event that the operator $\mathcal{A}$ depends on parameters q to be estimated, one then has data or "observations", say $z(t_i)$, and one attempts to choose a parameter $\bar{q}$ from an admissible set Q so as to yield a best fit of the model (1) to the data. For optimal control problems, the parameters q are presumed fixed and known and F in (1) is a control input term, say $F(t) = Bu(t)$. Then one has a *performance measure* J depending on z and u, and an admissible control set $\mathcal{U}$ (either open loop or feedback). One seeks a $\bar{u}$ in $\mathcal{U}$ that minimizes J subject to (1).

For either FDE or PDE systems, these problems involve infinite-dimensional state systems and hence computational schemes must be based on some type of approximation idea. The approach we describe here entails choosing a sequence of finite dimensional subspaces Z^N of Z generated by basis elements consisting of splines (linear, cubic, or quintic in the examples discussed below). One then approximates the system (1) (and the corresponding control or estimation problem) in the subspace Z^N by the system

$$\dot{z}^N(t) = \mathcal{A}^N(q)z^N(t) + P^N F(q,t)$$
$$z^N(0) = P^N z_0, \tag{2}$$

where P^N is the canonical orthogonal projection of Z onto Z^N and $\mathcal{A}^N \equiv P^N \mathcal{A} P^N$. This results in an approximate estimation or control problem entailing a finite dimensional state space to which standard computational packages can be applied.

The fundamental convergence theory which can be used in either control or parameter estimation problems is based on semigroup approximation results. Briefly the ideas are as follows (details differ depending upon whether one is treating FDE or PDE). One first demonstrates that $\mathcal{A}(q)$ satisfies a uniform dissipative inequality in Z (such as $\langle \mathcal{A}(q)z,z\rangle \le \omega \langle z,z\rangle$ for $z \in \mathrm{Dom}\,(\mathcal{A}(q))$) and $\mathcal{A}(q)$ (or some maximal dissipative extension) generates a C_0-semigroup $T(t;q)$. The approximating operators $\mathcal{A}^N(q)$ are defined, as we have already indicated, by $\mathcal{A}^N(q) = P^N \mathcal{A}(q) P^N$, and generate a stable family of schemes such that $|\exp[\mathcal{A}^N(q)t]| \le Me^{\omega t}$ where M and ω are independent of q and N. One uses standard estimates from spline approximation theory to argue that $\mathcal{A}^N(q)z \to \mathcal{A}(q)z$ in an appropriate sense. One then employs the Trotter-Kato theorem (a functional-analytic version of the Lax Equivalence theorem: stability plus consistency yield convergence) to establish that $\exp[\mathcal{A}^N(q)t] \to T(t;q)$ strongly in Z, and, moreover, that

$z^N(t;q) \to z(t;q)$ where z^N and z are solutions of (2) and (1), respectively. These convergence results are used in turn to argue existence of an estimate $\bar{q}=\lim \bar{q}^{N_k}$ for the parameter estimation problem involving (1), where $\{\bar{q}^{N_k}\}$ is some subsequence of $\{\bar{q}^N\}$, $\bar{q}^N$ a solution of the estimation probelm for (2). In the case of optimal control problems, one uses the convergence results to establish convergence of a subsequence $\bar{u}^{N_k}$ to $\bar{u}$, where $\bar{u}^N$ and $\bar{u}$ are solutions of the control problems for (2) and (1), respectively.

The necessary basic spline approximation theory for FDE can be found in (Banks and Kappel, 1979), whereas applications of the ideas to parameter estimation and control problems are given in (Banks, Burns, and Cliff, 1981), (Banks and Daniel, 1981a), (Daniel, 1982). Fundamental theory for PDE estimation and control problems is developed in (Banks and Kunisch, 1981) and (Banks, Crowley, and Kunisch, 1981). We shall not discuss further the theoretical aspects of our approach in this presentation; rather we shall briefly outline several specific problems in which use of these methods has been quite fruitful. The first application involves optimal feedback control for a delay system problem, while the second and third deal with parameter estimation problems for PDE.

FEEDBACK CONTROL FOR DELAY SYSTEMS

The problem of constructing feedback controls for hereditary or delay systems is not new and there is a rather large literature which we shall not discuss here. Our own renewed interest in this problem was motivated by problems arising in the design of controllers for a liquid nitrogen wind tunnel (the National Transonic Facility or NTF) currently under construction by NASA at its Langley Research Center in Hampton, VA. With this wind tunnel it is expected that researchers will be able to achieve an order of magnitude increase in the Reynolds number over that in existing tunnels while maintaining reasonable levels of dynamic pressure. Test chamber temperatures (the Reynolds number is roughly inversely proportional to temperature) will be maintained at cryogenic levels by injection of liquid nitrogen as a coolant into the airstream near the fan section of the tunnel. In addition to a gaseous nitrogen vent to help control pressure, motor driven fans will be used as the primary regulator of Mach number. Fine control of Mach number will be effected through changes in inlet guide vanes in the fan section. Schematically, the tunnel can be depicted as in Fig. 1.

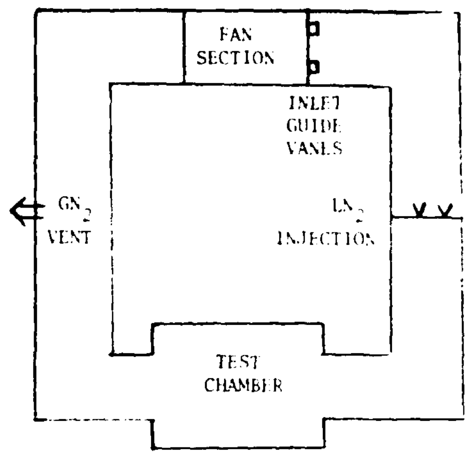

Fig. 1.

The basic physical model relating states such as Reynolds number, pressure, and Mach number to controls such as LN_2 input, GN_2 bleed, and fan operation involves a formidable set of PDE (the Navier-Stokes theory) to describe fluid flow in the tunnel and test chamber. This model has, not surprisingly, proved to be very unwieldy from a computational viewpoint and is difficult, if not impossible, to use directly in the design of sophisticated control laws. (Both open loop and feedback controllers are needed for efficient operation of the tunnel - and this is a desirable goal since cost estimates for liquid nitrogen alone are $\$6.5 \times 10^6$ per year of operation.) In addition to the design of both open loop and closed loop controllers, parameter estimation techniques will be useful once data from the completed tunnel is available (current investigations involve use of data from a 1/3 meter scale model of the tunnel).

In view of the schematic in Fig. 1, it is not surprising that engineers (e.g. see (Armstrong and Tripp, 1981) and (Gumas, 1979)) have proposed design of control laws for subsystems modeled by lumped parameter models (the variables represent values of states and controllers at various discrete locations in the tunnel and test chamber) with transport delays to account for flow times in sections of the tunnel. A specific example is the model (Armstrong and Tripp, 1981) for the Mach no. control loop in which variations in the Mach no. (in the test chamber) are, to first order, controlled by variations in the inlet guide vane angle setting (in the fan section) - i.e. $\delta M(t) \sim \delta\theta(t-\tau)$ where τ represents a transport time from the fan section to the test section. More precisely, the proposed equation

relating the variation δM (from steady state operating values) in Mach no. to the variation $\delta\theta$ in guide vane angle is

$$\tau\delta\dot{M}(t) + \delta M(t) = k\,\delta\theta(t-r)$$

while the equation relating the guide vane angle variation to that $\delta\theta_A$ of an actuator is

$$\delta\ddot{\theta}(t) + 2\zeta\omega\delta\dot{\theta}(t) + \omega^2\delta\theta(t) = \omega^2\delta\theta_A(t).$$

Rewriting the system in vector notation, one thus finds that the Mach no. control loop involves a regulator problem for the equation

$$\dot{x}(t) = A_0x(t) + A_1x(t-r) + B_0u(t) \qquad (3)$$

where $x = (\delta M, \delta\theta, \delta\dot{\theta})$, $u = \delta\theta_A$. Here the control is the guide vane angle actuator input. A similar 4-vector system problem can be formulated in the case where one treats the actuator rate $\delta\dot{\theta}_A$ as the control - see (Armstrong and Tripp, 1981), (Daniel, 1982).

Problems such as that just outlined led us to consider the spline techniques of (Banks and Kappel, 1979) for computation of feedback controls in regulator problems governed by n-vector systems

$$\begin{aligned} \dot{x}(t) &= L(x_t) + Bu(t) \\ (x(0), x_0) &= (\Phi(0), \Phi) \end{aligned} \qquad (4)$$

where x_t denotes the function $s \to x(t+s)$, $-r \le s \le 0$, and $L(x_t) = \sum_{j=0}^{\nu} A_jx(t-r_j) + \int_{-r}^{0} A(s)x(t+s)ds$ with $0 = r_0 < r_1 < \ldots < r_\nu = r$. The cost functional is the usual integral quadratic payoff

$$J(z_0,u) = J((x(0),x_0),u) = \int_0^\infty x(t)D_0x(t) + u(t)Ru(t)$$

where $t \to x(t)$ is the solution of (4). As is well-known (see the summary and references to previous literature in (Gibson, 1980)) the appropriate state feedback control is given in terms of a functional

$$u(x_t) = -R^{-1}B^T[K_0x(t) + \int_{-r}^{0} K_1(s)x(t+s)ds] \qquad (5)$$

where the gains K_0, K_1 satisfy certain Riccati type equations. A detailed explanation of use of the spline-based methods for computations in these problems is given in (Banks and Rosen, 1982); we only outline the procedures here and discuss our numerical findings for the NTF example.

Briefly then, one reformulates the system (4) as an abstract system (1) in the Hilbert space $Z = R^n \times L_2(-r,0;R^n)$ with $z(t) = (x(t),x_t)$. The optimal feedback control is then given by (see the summary in §4 of (Gibson, 1980))

$$\bar{u}(t) = -R^{-1}\mathscr{B}^*\Pi z(t) \qquad (6)$$

with $\bar{J} = J(z_0,\bar{u}) = \langle \Pi z_0, z_0\rangle$ where the bounded linear operator $\Pi: Z \to Z$ is the solution of the Riccati algebraic equation (RAE):

$$\mathscr{A}^*\Pi + \Pi\mathscr{A} - \Pi\mathscr{B}R^{-1}\mathscr{B}^*\Pi + \mathscr{D} = 0.$$

Here $\mathscr{B}$ and $\mathscr{D}$ are operators on Z given by $\mathscr{B}(\eta,\phi) = (B\eta,0)$ and $\mathscr{D}(\eta,\phi) = (D_0\eta, 0)$. Carrying out the approximation steps outlined above (we used the first order spline subspaces explained in detail in §4, p.509-512 of (Banks and Kappel, 1979) for the calculations on the NTF example and a number of other test examples reported on in (Banks and Rosen, 1982)) one obtains an approximating RAE corresponding to the associated regulator problem for (2) in Z^N. This is given by

$$\mathscr{A}^{N*}\Pi_N + \Pi_N\mathscr{A}^N - \Pi_N\mathscr{B}^NR^{-1}\mathscr{B}^{N*}\Pi_N + \mathscr{D}^N = 0$$

where $\mathscr{A}^N = P^N\mathscr{A}P^N$, $\mathscr{B}^N = P^N\mathscr{B}$, $\mathscr{D}^N = P^N\mathscr{D}P^N$. This equation has a matrix representation and hence standard techniques - we used the ORACLS package of Armstrong (1980) in our calculations - can be employed to calculate Π_N and the resulting approximate optimal feedback control

$$\bar{u}^N(t) = -R^{-1}B^T[\Pi_N^{00}x(t) + \int_{-r}^{0}\Pi_N^{10}(s)x(t+s)ds] \qquad (7)$$

where Π_N^{00}, Π_N^{10} are components of Π_N which approximate the gains K_0, K_1 of (5).

We investigated the Mach no. control loop problem of (Armstrong and Tripp, 1981) described above with the spline-based methods and with the so-called "averaging" approximation method (a "zero-order" spline type scheme which can also be developed in the context of Walsh function approximations - that is not of the form $\mathscr{A}^N = P^N\mathscr{A}P^N$ but nonetheless can be used to compute gains in delay system problems - for details see (Gibson, 1980)). For this example the matrices in (3) have the form

$$A_0 = \begin{bmatrix} -1/\tau & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega^2 & -2\zeta\omega \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & k/\tau & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 \\ 0 \\ \omega^2 \end{bmatrix}$$

with the parameter values given by $\tau = 1.964$ sec, $r = .33$ sec, $\omega^2 = 36$, $\zeta = .8$, $k = -.0117$.

Calculations were carried out for the problem of driving δM from $-.1$ to 0.0 (corresponding to M varying from .8 to .9) and $\delta\theta$ from 8.55 to 0.0 (corresponding to the guide vane angle varying from 10.48° to a steady state of 1.93°). Excellent results were obtained even for low values of the approximation index N (N = 2,4,8). The corresponding optimal controls (7) appear to converge rapidly to an optimal control of the form (6) (of course, we

do not know Π for this example) and when used as feedback in (3), yield trajectories as graphed in Fig. 2. (In Fig. 2 we compare these with similar trajectories obtained in (Armstrong and Tripp, 1981) where finite difference techniques - assuming piecewise constant controls - were used to discretize the delay system before applying standard regulator theory for discrete systems to calculate the feedback controllers.)

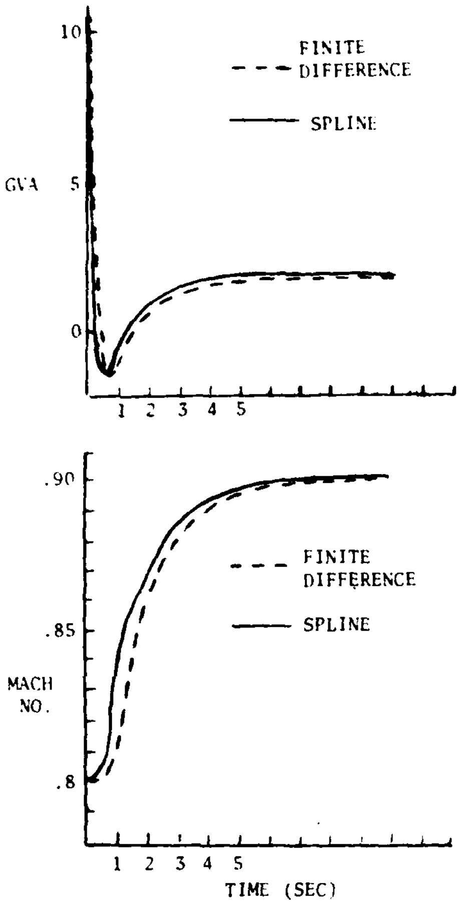

Fig. 2.

The spline method was computationally more stable and in general more accurate than the averaging method. Both methods appear to offer higher order approximations than the finite difference techniques of (Armstrong and Tripp, 1981). All three methods appear to be adequate for the simple NTF example we investigated.

We also tested the spline method (and compared it with the averaging method) on a number of other delay system regulator examples and our findings are detailed in (Banks and Rosen, 1982). In summary we found the spline method generally at least as good as and in some cases superior to the averaging method with regard to *computational stability*, accuracy, and rate of convergence. We refer interested readers to (Banks and Rosen, 1982) for detailed remarks on the computations along with some comments on theoretical aspects (see also (Gibson, 1980) and (Kunisch, 1980) for *related discussions*).

PARAMETER ESTIMATION IN POPULATION DISPERSAL MODELS

An important problem to population ecologists involves modeling of dispersal or movement of insects and other herbivores in vegetation patches. There is a growing literature (Levin, 1974, 1981), (Okubo, 1980), (Kareiva, 1982) on the use of mathematical models, especially ones entailing distributed systems, to investigate the effect of various types of transport mechanisms on overall population movement. Many important ecological investigations result in the need to solve an inverse problem for a diffusion or more general transport equation. Roughly speaking, given data describing changing population densities of a specie (or species) and some preliminary hypotheses about boundary conditions and transport functions (spatially, temporally or perhaps even density dependent) in a proposed model, one desires to estimate or identify parameters (including the transport functions) in the model and quantify the success (or lack thereof) of the model in describing the data. For example, typical models might involve the general transport equation (Okubo, 1980, p.98) (in one spatial variable) for population density u (here we mention only single specie models but coupled equations for multiple species models could also be treated with the ideas we outline) given from mass balance considerations by

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(Vu) = \frac{\partial}{\partial x}\left(\mathcal{D}\frac{\partial u}{\partial x}\right) + f(x,u) \qquad (8)$$

where the "directed movement" or advective (convective) term $\frac{\partial}{\partial x}(Vu)$ contains a spatially varying "velocity" term $V = V(x)$. The diffusion term is, as usual, a result of assuming Fick's first law of diffusion, while f represents a general birth/death term. In such models it is often important to allow the transport functions $\mathcal{D}$ and V to vary spatially, temporally, or even with population density (or perhaps some combination of these). Other basic transport assumptions or hypotheses (e.g., see (Okubo, 1980, p.84-88), (Dobzhansky and colleagues, 1979)) lead to different models, but in most cases a very important problem consists of using field data to estimate the transport functions (such as $\mathcal{D}$ and V) and perhaps birth/death parameters in f.

We have successfully applied the spline methods outlined above in connection with (1) and (2) to such problems (Banks and Kareiva, 1982). In addition to (Banks and Kareiva, 1982), one may consult (Banks, 1981), (Banks, Crowley, and Kunisch, 1981) for the theory behind our efforts. Briefly, one rewrites (8) in the form (1) in the Hilbert space $Z = L_2(0,1)$ and then uses the approximating equation (2) - in this case we employed cubic splines

for the basis elements in Z^N - with the data to estimate the unknown transport functions. For the resulting finite dimensional problems we employed a standard IMSL package (ZXSSQ) for the Levenberg-Marquardt algorithm in our parameter search for a fixed level of approximation N. In the particular problems we investigated, we hypothesized equation (8) in which $V = V(x)$, $\mathcal{D}$ is constant, and f contains piecewise linear (in u) terms with spatially dependent coefficients. We also hypothesized unknown parameters in the initial population density. Our early efforts with field data collected by P. Kareiva (the experiments involved the dispersal of flea beetles in cultivated collard patches) revealed that models such as (8) with a spatially dependent V yield significantly better fits to the data than do models with V vanishing or chosen as some nontrivial constant. Our more recent efforts (detailed in (Banks and Kareiva, 1982)), again using the flea beetle data, involve the particular equation (obtained from (8) after some transformations and assumptions)

$$\frac{\partial u}{\partial t} = q_1 \frac{\partial^2 u}{\partial x^2} + q_2(x)\frac{\partial u}{\partial x} + q_3(x)u + g(t,x) \qquad (9)$$

for $t > 0$, $0 \le x \le 1$. The function q_2 is assumed to have the form

$$q_2(x) = \begin{cases} -\gamma(x-.5) & .5-L \le x \le .5+L \\ 0 & \text{otherwise,} \end{cases}$$

where $\gamma < 0$, *and* q_3 *contains an appropriate* death rate term (modest death rates within the vegetation patch, high death rate outside the patch) in addition to a term involving q_2'. The function g contains terms arising from standard transformations of (8) with nonhomogeneous boundary conditions to (9) with homogeneous conditions. The cubic spline-based estimation techniques have performed extremely well in our efforts to estimate q_1, q_2, q_3 as well as the initial conditions from the data. The methods were very stable and rapidly convergent, yielding satisfactory estimates for low ($N = 8,16$) values of the approximation index.

There is strong evidence (Dobzhansky and colleagues, 1979),(Aikman and Hewitt, 1972) of the need in certain population studies to estimate time dependent transport coefficients. Our cubic spline methods can be developed for these problems (see (Banks and Daniel, 1981b) for preliminary theoretical results) and we *are currently pursuing investigations along* these lines.

There are also numerous important control problems arising in the context of ecological investigations. Once an adequate model is developed (the parameter estimation problem), one might wish to estimate (calculate) the *optimal vegetation density* in a patch in order to hold population levels in the patch to a minimum, or at least below some given level. We believe that the methods discussed here will also prove useful in these problems.

PARAMETER ESTIMATION IN ELASTIC STRUCTURES

We turn finally to a brief discussion of use of cubic and quintic spline schemes for parameter estimation problems arising in the study of elastic and viscoelastic bodies. Our interest in such problems was motivated by discussions with NASA engineers who desire to estimate material properties for large space structures from observations of the motions of these structures. The simplest components of these antennae and space stations are beam-like and made from composite materials (e.g. graphite epoxy). Thus as a basic problem, we (Banks and Crowley, 1981) considered estimation in equations such as those arising in the Euler-Bernoulli theory for transverse vibrations of a thin elastic or viscoelastic beam which is possibly subject to damping. More precisely, the well-known equations for the transverse vibrations of a thin elastic beam (no damping) are

$$\mathcal{M} = EI\frac{\partial^2 u}{\partial x^2}$$

$$m\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 \mathcal{M}}{\partial x^2} = f(q,t,x)$$

where $\mathcal{M}$ is the bending moment, m is the mass per unit length and f is the applied load. Two types of damping are included in our formulation. The first is simply viscous damping γu_t while the second is structural damping. For a Voight solid (the simplest viscoelastic model) one has the constitutive relationship $\sigma = E\varepsilon + c\dot{\varepsilon}$. Thus the stress σ is no longer proportional to the strain alone (as in Hooke's law) but a term proportional to the strain velocity is added. In this case the usual Euler-Bernoulli formulation becomes

$$\mathcal{M} = \int \sigma y\, dA = EI\frac{\partial^2 u}{\partial x^2} + cI\frac{\partial^3 u}{\partial x^2 \partial t} .$$

This results in the equation

$$mu_{tt} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 u}{\partial x^2} + cI\frac{\partial^3 u}{\partial x^2 \partial t}\right) + \gamma u_t = f, \quad (10)$$

which can be rewritten in the form (1) in the Hilbert space $Z = H^2 \times L_2$. We have developed and tested estimation schemes (for estimation of parameters such as EI/m, cI/m, γ/m) using cubic and quintic spline approximation subspaces Z^N modified to treat various important boundary conditions (simply supported, cantilevered, as well as beams with applied moments at one end). The methods proved extremely efficient as the detailed presentation in (Banks and Crowley, 1981) documents.

While the Euler-Bernoulli equation (10) is applicable in many applications (especially large space structures), a somewhat more

involved analysis is required in situations where rotatory inertia and shear effects play a significant role in the dynamics. This theory is often necessary when high frequency oscillations of the beam must be considered (e.g. in aerodynamic structures). In this event the Timoshenko formulation is more appropriate. This theory can be embodied in a single higher order equation (fourth order in t and x derivatives) where the boundary conditions for even the simply supported beam involve second order derivatives in both x and t. For our purposes it is much more desirable to treat a system of lower order equations with the corresponding boundary conditions. The equations modeling transverse vibrations of a homogeneous isotropic elastic beam, including rotatory inertia and shear effects, can be written in terms of the transverse displacement y and the angle ψ of rotation of the beam cross section from its original vertical position as

$$
\begin{aligned}
y_{tt} &= a^2(y_{xx} - \psi_x) \\
\psi_{tt} &= b^2\psi_{xx} + c^2(y_x - \psi)
\end{aligned}
\tag{11}
$$

with the boundary conditions for, say, a fixed end beam given by $y(t,0) = y(t,l) = 0$, $\psi(t,0) = \psi(t,l) = 0$. Here $a^2 = k'AG/m$, $b^2 = EI/m$, $c^2 = Aa^2/I$ with A = cross sectional area, E = Young's modulus, G = shear modulus, I = moment of inertia, and k' = *shear coefficient.*

Equation (11) can be rewritten in the form (1) in the Hilbert space $Z = H_0^1 \times L_2 \times H_0^1 \times L_2$ and then cubic spline schemes can be applied (the approximating equations again have the form (2)) to estimate parameters such as a, b, and c. We did this (Banks and Crowley, 1981) and once again extremely efficient algorithms resulted in very accurate estimates.

CONCLUSION

We have outlined above several problems to which our spline based approximation techniques can be applied with great success. Both theoretical and numerical findings (some reported in the literature cited, some as yet unreported in manuscripts) support our claim that these methods have even wider applicability than we have indicated here. For example, we are currently successfully applying the methods for estimation of parameters in nonlinear FDE (Banks and Daniel, 1981a) to the study of models for the enzyme column reactors as discussed in (Banks, 1981), (Daniel, 1981). As one might anticipate from the elasticity examples mentioned above, both the theoretical soundness and computational feasibility of our methods have been demonstrated for hyperbolic systems. In particular, we have successfully developed the theory and computational packages to treat test problems in seismic inversion (see (Banks, 1981)) in which not only the parameter E in $u_{tt} = (Eu_x)_x$, but also parameters k_1, k_2 in elastic $(u_x(t,0) + k_1u(t,0) = 0)$ and absorbing $(u_t(t,l) + k_2u_x(t,l) = 0)$ boundary conditions must be identified from data.

A general theory plus numerical results obtained when applying our approximation methods to nonlinear hyperbolic and parabolic PDE can be found in (Banks and Kunisch, 1982), (Banks, Crowley, and Kunisch, 1981). Other areas of application in which we have already used or are currently using these spline based methods include estimation problems for transport of labelled substances in brain tissue, determination of static antenna configuration and shape, and estimation of porosity and permeability in porous media.

ACKNOWLEDGEMENTS

Work reported here was supported in part by the Air Force Office of Scientific Research under contract 81-0198, in part by the National Science Foundation under grant NSF-MCS 7905774-05, and in part by the U. S. Army Research Office under contract ARO-DAAG29-79-C-0161. Parts of the research discussed were carried out while the author was a visitor at the Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, VA, which is operated under NASA contracts No. NAS1-15810 and No. NAS1-16394.

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